Recognizing regular tree languages with static information

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Motivation

- ▷ Efficient compilation of patterns in XDuce/CDuce/. . .
- ▷ E.g.:

```
type A = <a>[ A* ]
type B = <b>[ B* ]
let f ((A|B) -> Int) A    -> 0 | B -> 1
let g ((A|B) -> Int) <a>_ -> 0 | _ -> 1
```

- ▷ Encourage more declarative style in patterns.

The separation problem

- D: domain of values

  Given $X_1, \ldots, X_n \subset D$ (pairwise disjoint) and $v \in \bigcup X_i$, returns i s.t. $v \in X_i$

- Two-stage process: the X_i come first (compile-time), and v comes after (run-time)

  Given $X_1, \ldots, X_n$, computes a function $s : \bigcup X_i \to \{1, \ldots, n\}$ s.t. $\forall v \in \bigcup X_i,\ v \in X_{s(v)}$

- We want s to be efficient, even if computing it is expensive.
- Parameters: the domain D, the class of possible X_i.
- Note: $\bigcup X_i$ is the *static information* provided by the type system.

Examples

- D: integers; X_i: unions of intervals.
  - Decision tree (dichotomy)
- D: strings; X_i: finite sets.
  - Trie.
  - Can ignore suffixes.
  - E.g.: $X_1 = \{aa, ab, ac\}, X_2 = \{ba, bb, bc\}$
- D: strings; X_i: regular languages.
  - Deterministic automaton.
- D: XML documents; X_i: XML types (schema) = regular tree languages.
  - ???

Expected properties

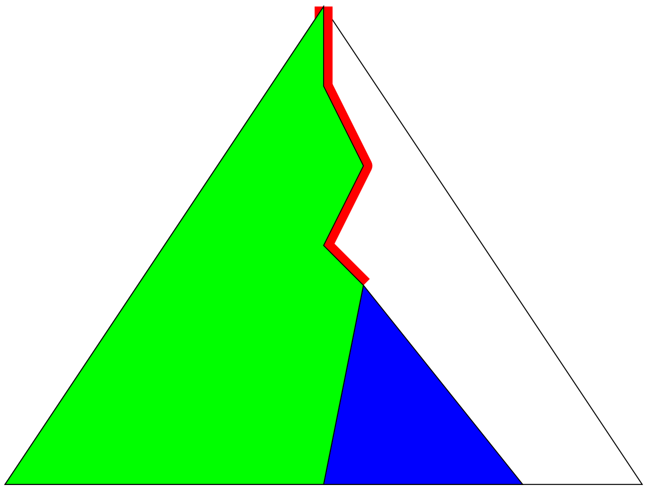
- Traverse each node of the tree only a finite bounded number of times.
- Ignore subtrees that can be ignored
  - because of the static information and/or
  - because they are irrelevant.
- Independance w.r.t. the syntax of types (only the denoted sets matter).

Classical solutions

- Backtracking tree automata
  - Don’t guarantee linear time recognition.
- Bottom-up deterministic tree automata.
  - Only consider downward context (subtree).
  - Ignore upward context (path).
  - Cannot adapt their behavior to the current location in the tree.
- Top-down deterministic tree automata
  - Cannot recognize arbitrary regular tree languages.
  - Only consider the upward context.
  - No left-to-right propagation of information.

Contexts

- Path
- Subtree
- Left



Trees:

$$v \quad ::= \quad a \mid (v_1, v_2)$$

(ie: $V = \Sigma + V \times V$)

Regular languages defined by equations:

$$\begin{cases} X_1 = \Sigma_1 \cup X_{\dots} \times X_{\dots} \cup \ldots \cup X_{\dots} \times X_{\dots} \\ \ldots \\ X_n = \Sigma_n \cup X_{\dots} \times X_{\dots} \cup \ldots \cup X_{\dots} \times X_{\dots} \end{cases}$$

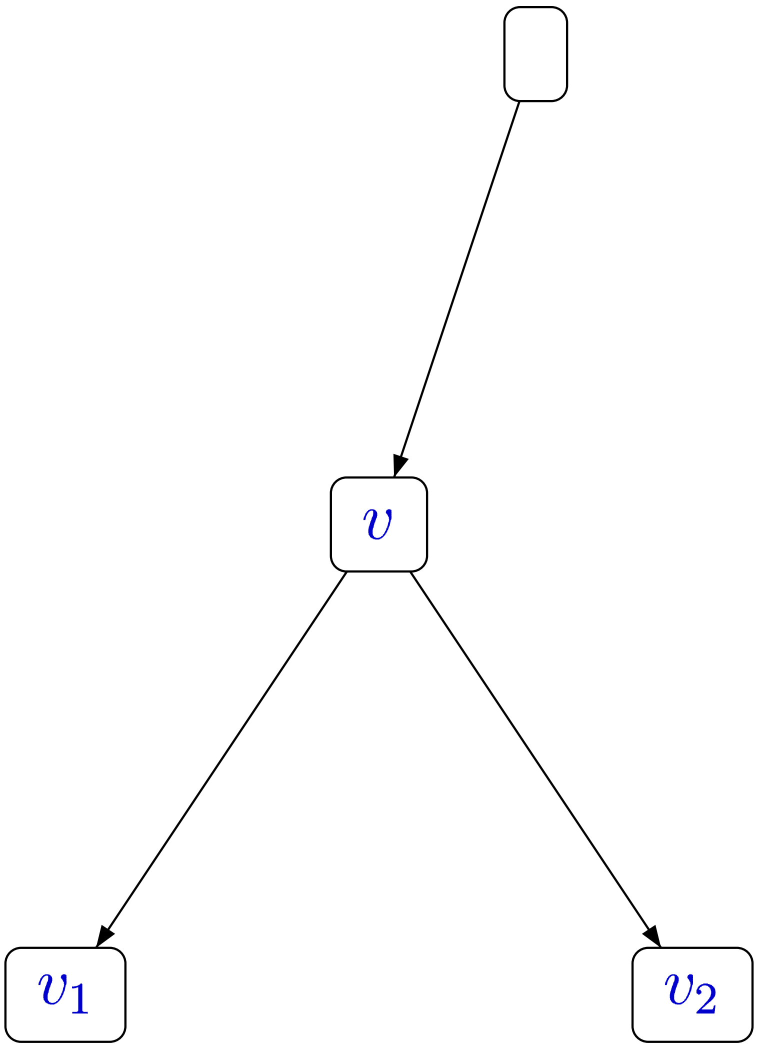
E.g.:

$$\begin{cases} X_0 = X_1 \times X_2 \\ X_1 = \{a\} \cup X_1 \times X_1 \\ X_2 = \{b\} \cup X_2 \times X_2 \end{cases}$$

Non-uniform automata

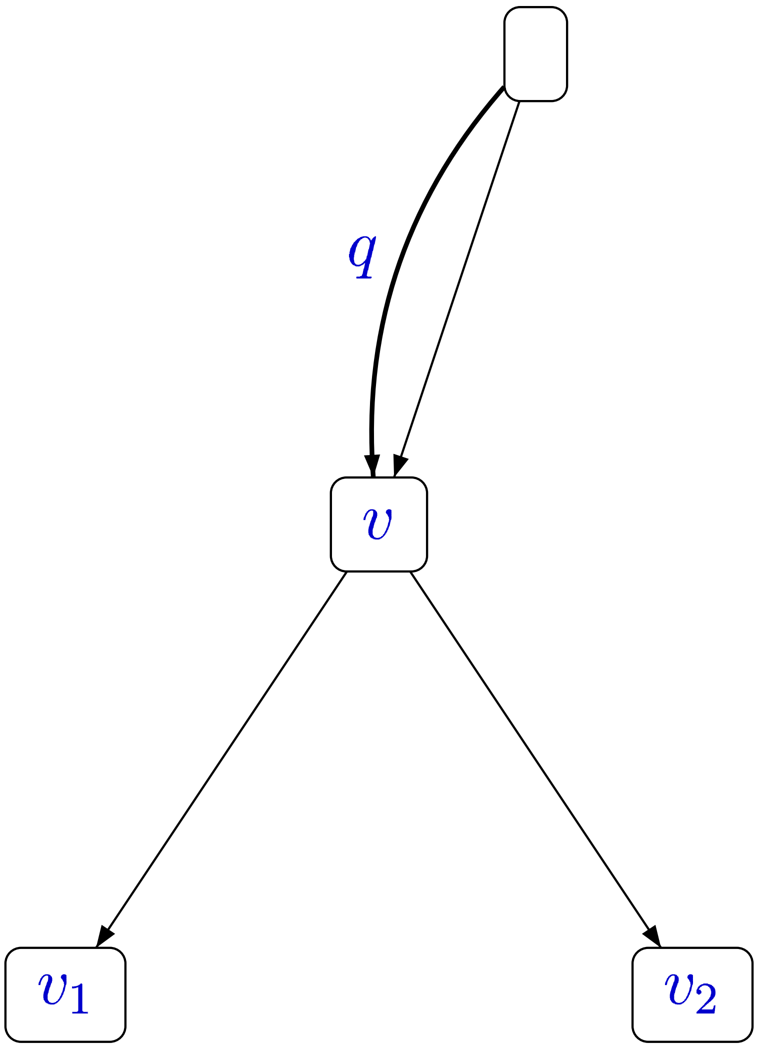
- A specific kind of push-down automata.
  - Combine the advantages of bottom-up and top-down TA.
  - Can totally ignore whole subtrees.
- Fixed traversal order.
- Thread a control-state through the traversal.
  - Accumulates knowledge gained from the traversal.
  - Depends on the left context (not only path).

Non-uniform automata

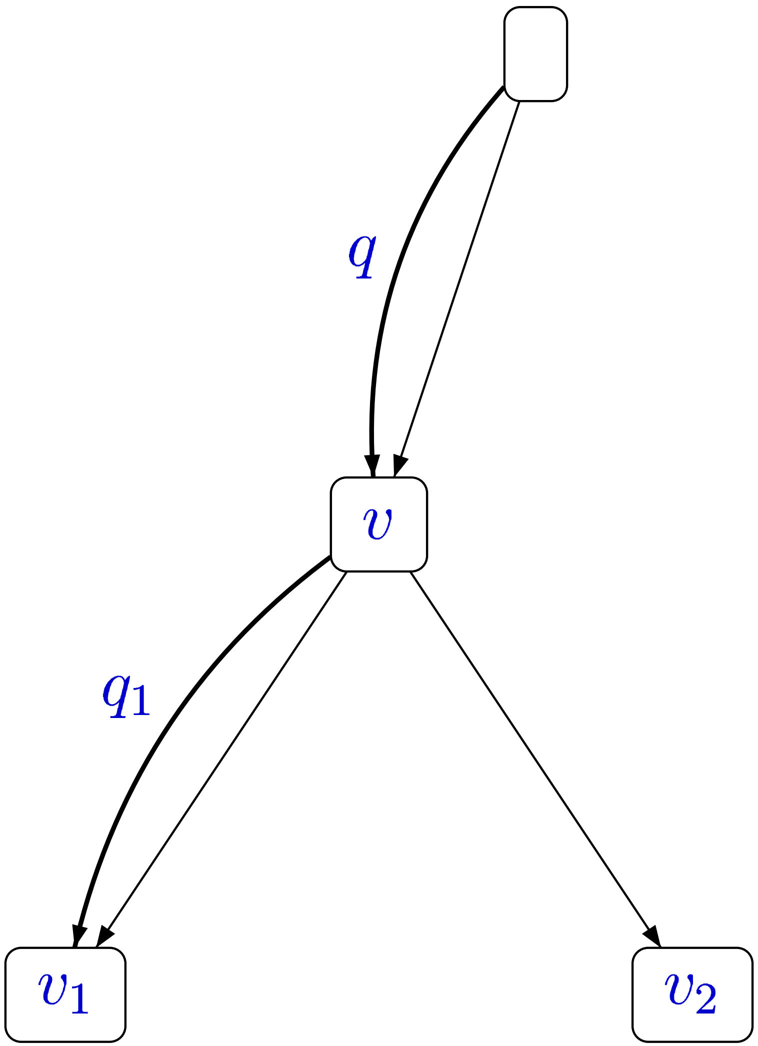


Non-uniform automata

Input: state q



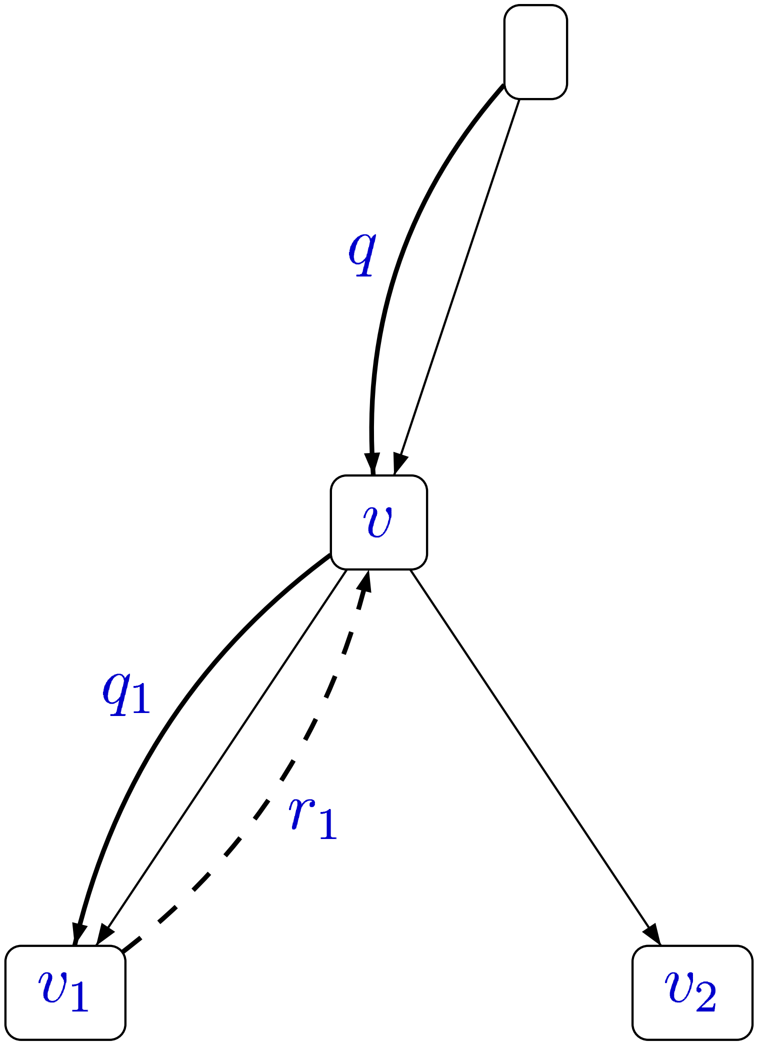
Non-uniform automata



Input: state q

$q \mapsto q_1$

Non-uniform automata

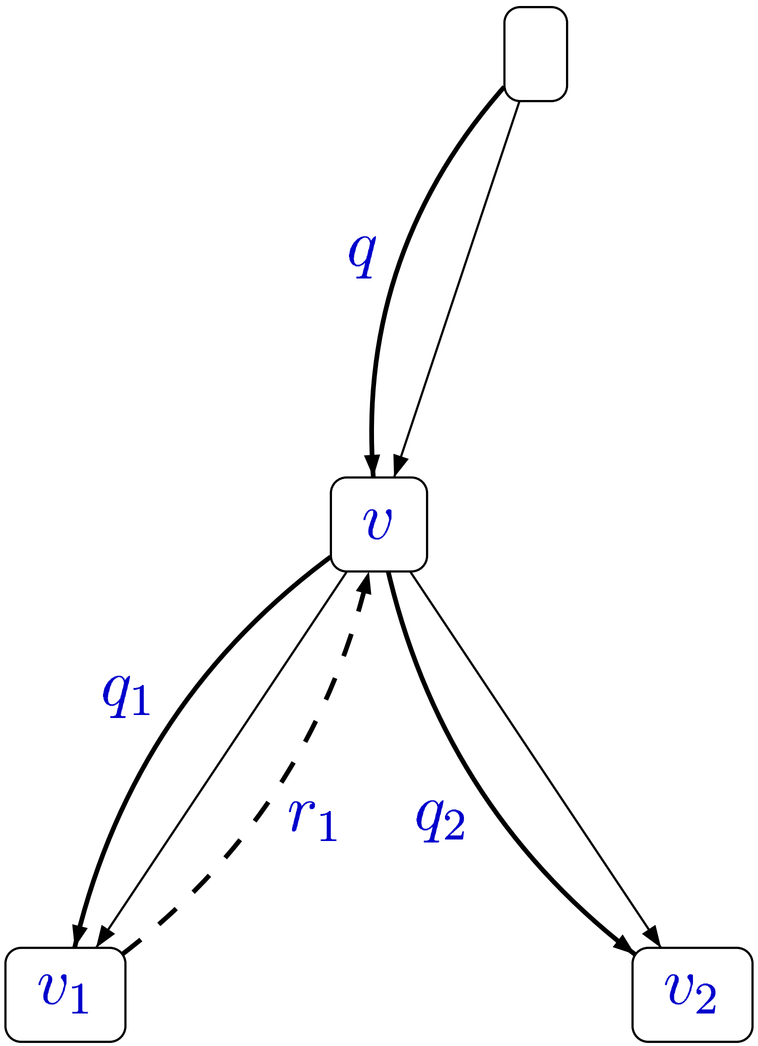


Input: state q

$q \mapsto q_1$

$r_1 \in R(q_1)$

Non-uniform automata



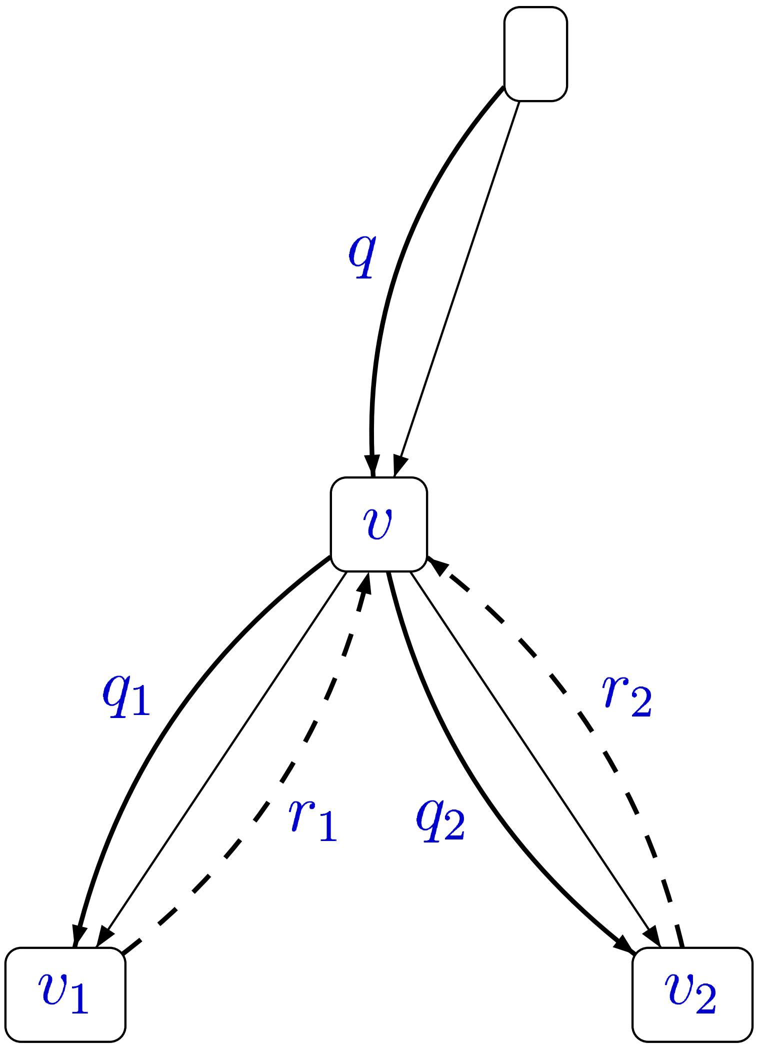
Input: state q

$q \mapsto q_1$

$r_1 \in R(q_1)$

$q, r_1 \mapsto q_2$

Non-uniform automata



Input: state q

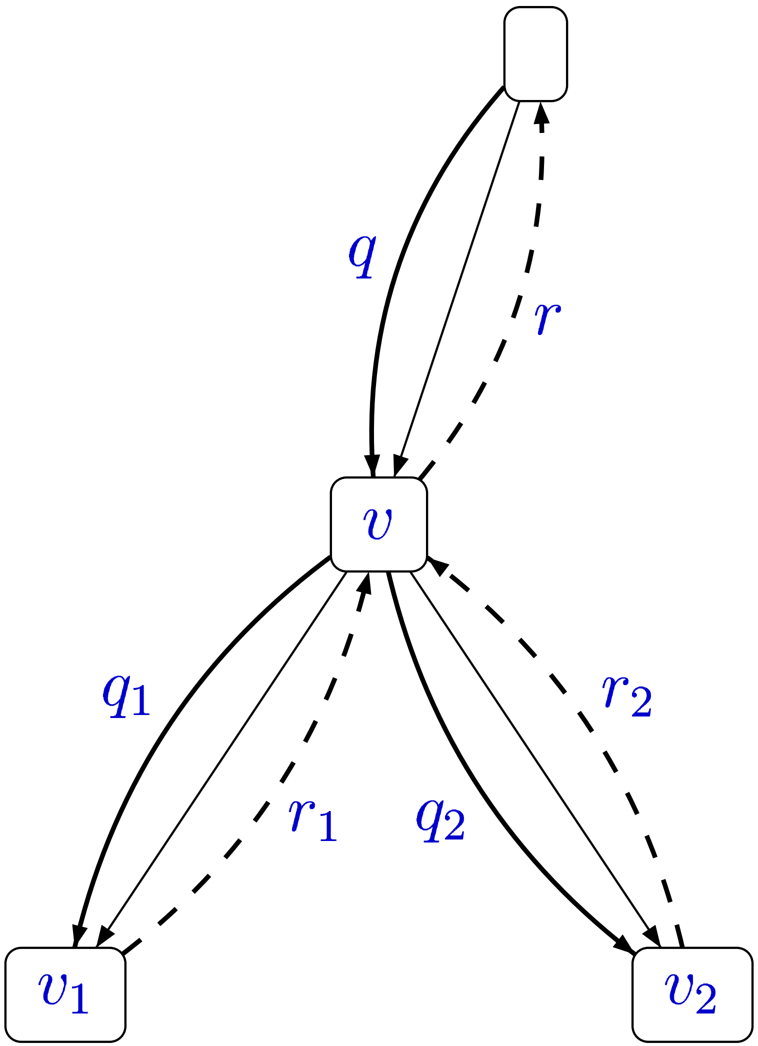
$q \mapsto q_1$

$r_1 \in R(q_1)$

$q, r_1 \mapsto q_2$

$r_2 \in R(q_2)$

Non-uniform automata



Input: state q

$q \mapsto q_1$

$r_1 \in R(q_1)$

$q, r_1 \mapsto q_2$

$r_2 \in R(q_2)$

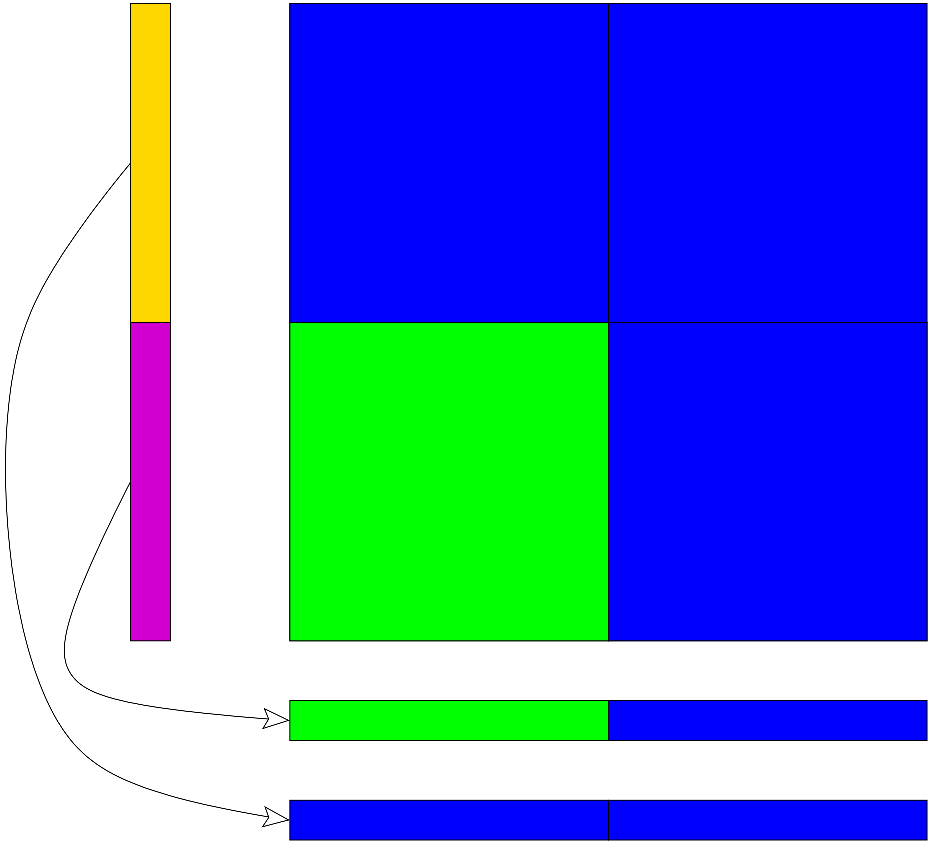
$q, r_1, r_2 \mapsto r \in R(q)$

Compilation

- Main technical contribution: a compilation algorithm that generates efficient NUA (which ignore many subtrees).
- Key idea: propagate static information precisely (in the control state)

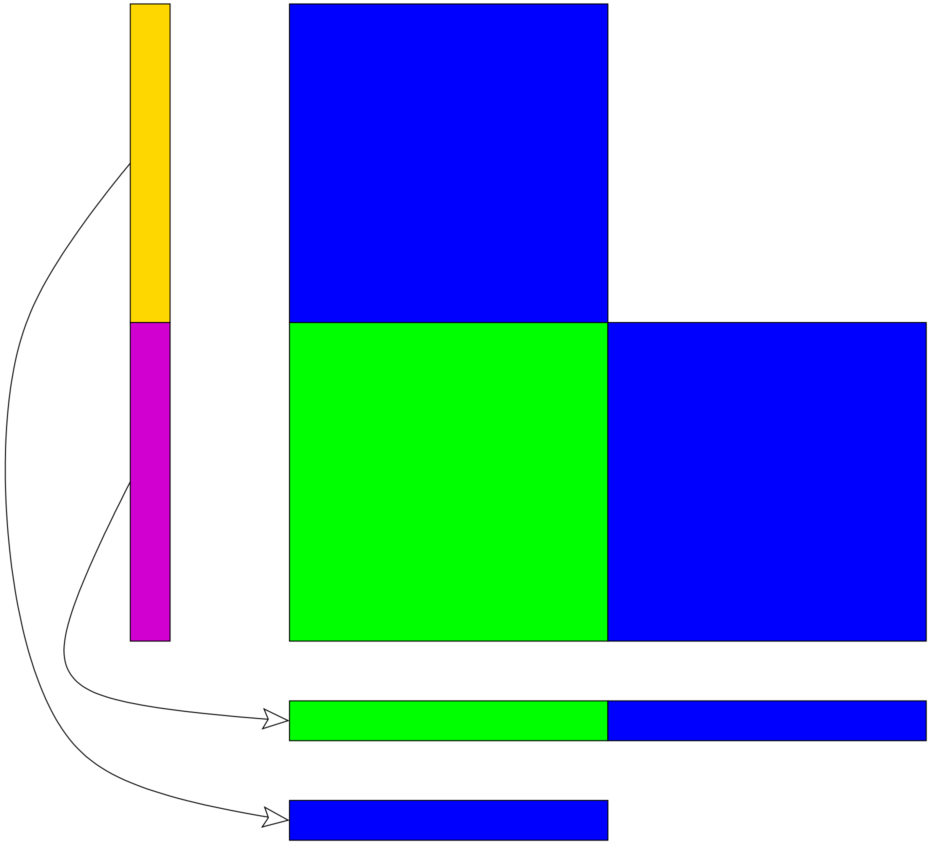
Compilation

- May ignore right subtree, according to left subtree.



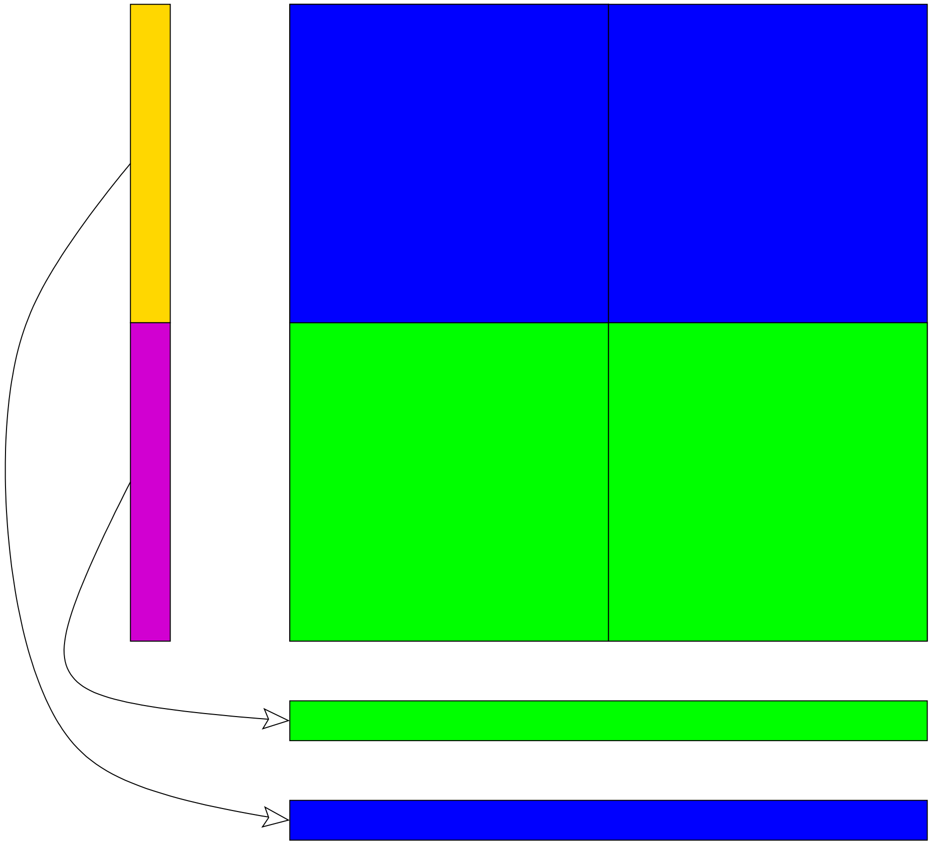
Compilation

▷ Propagate static information.



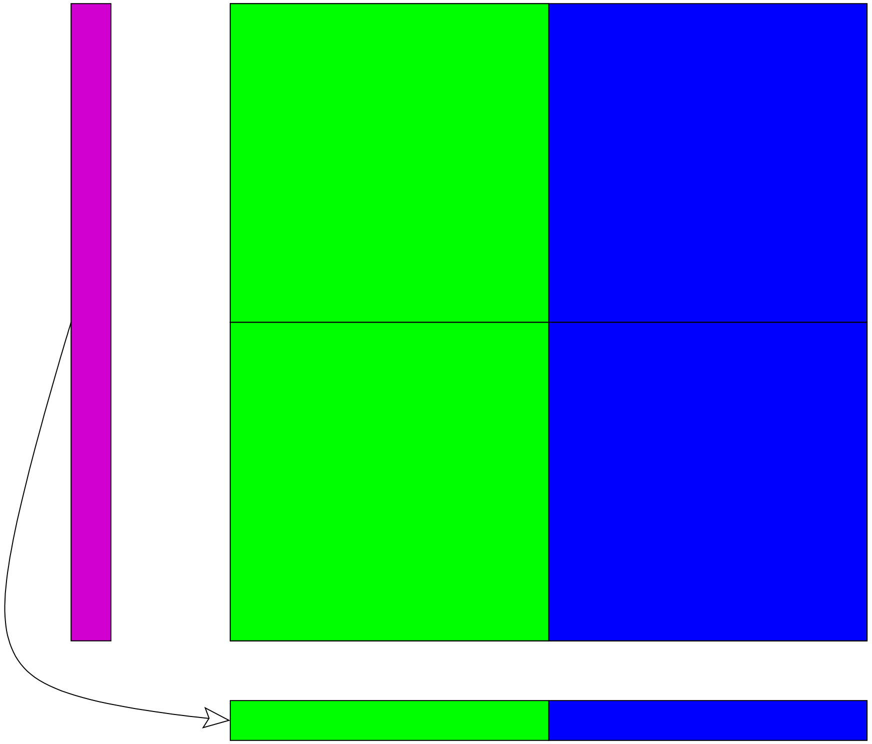
Compilation

- Ignore right subtree.



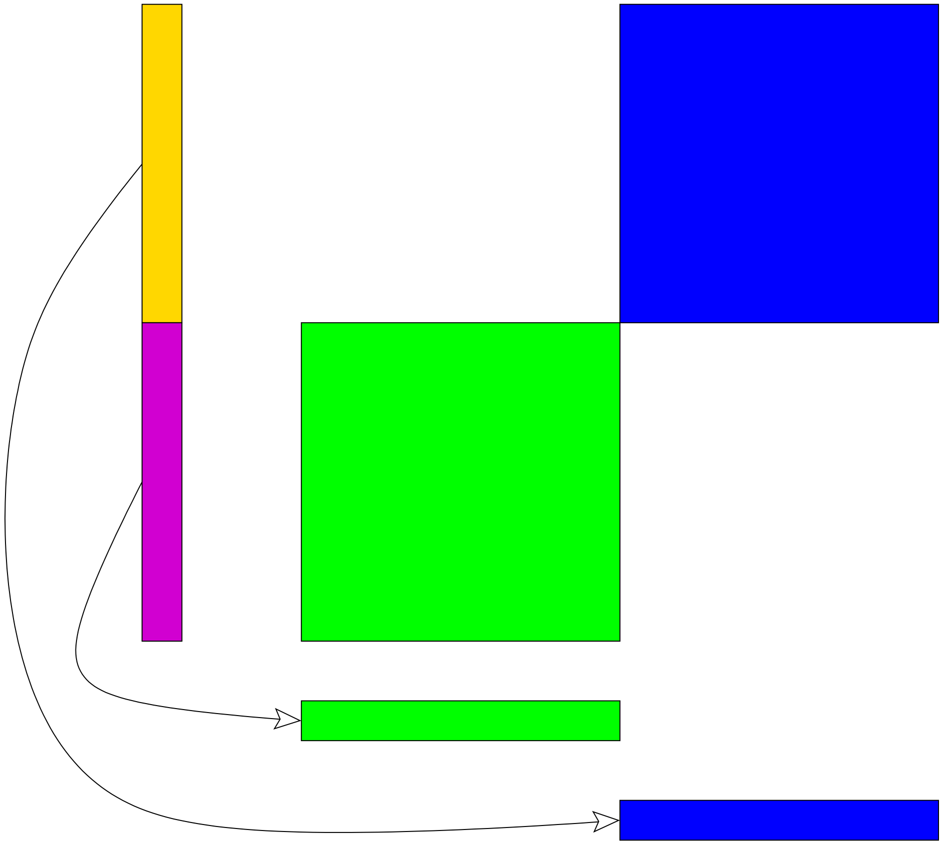
Compilation

- ▷ Ignore left subtree (normalization).



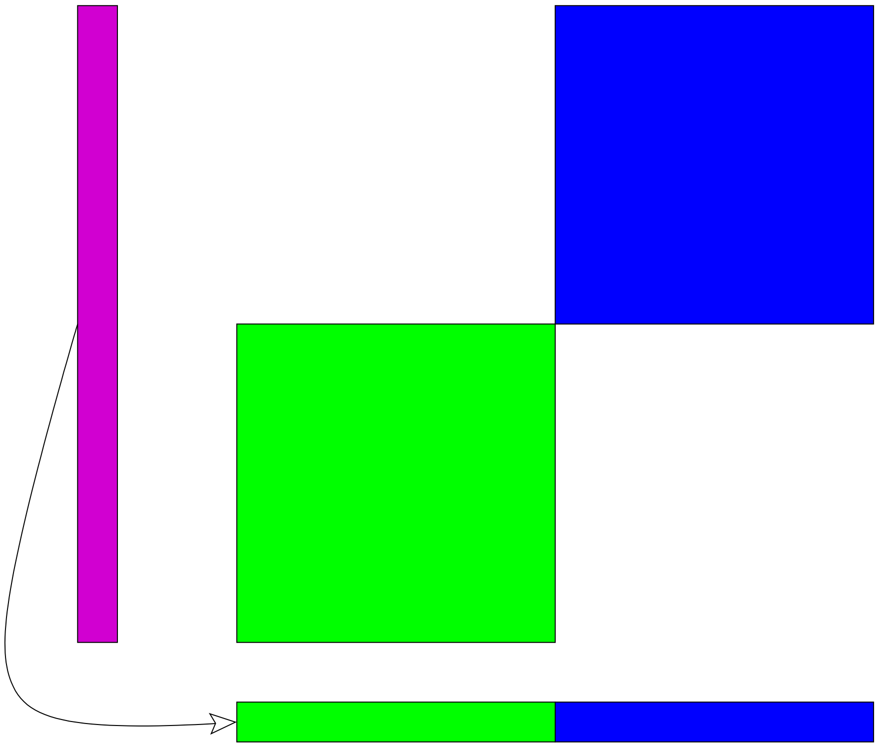
Compilation

- Ignore right subtree.



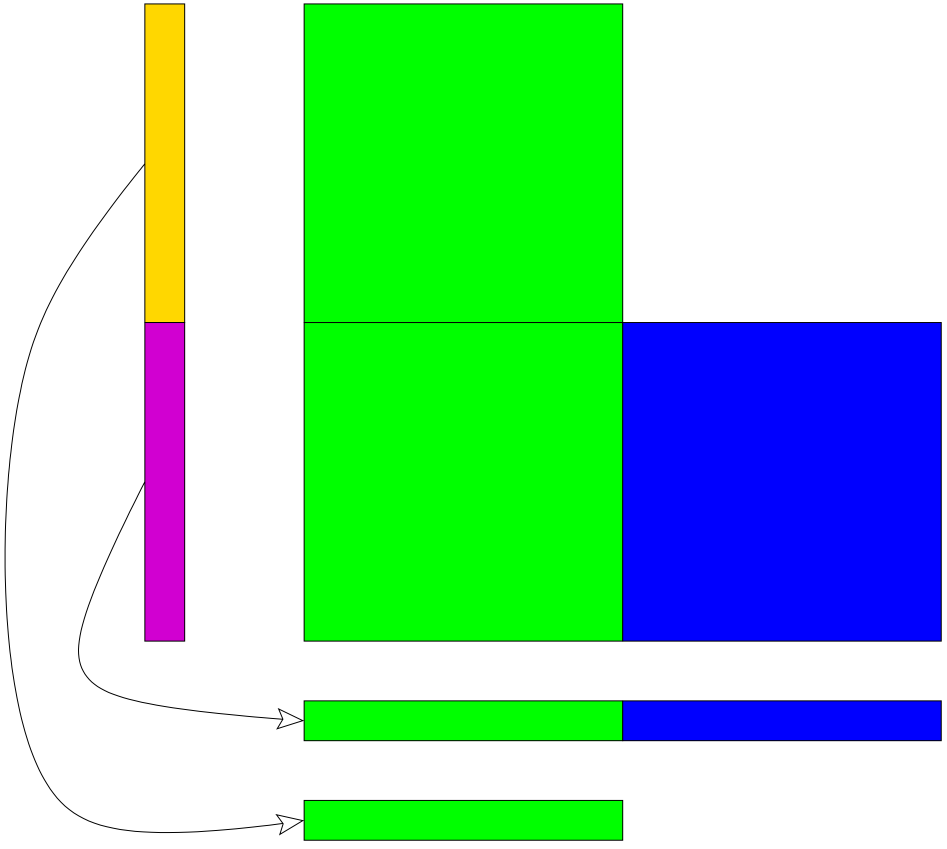
Compilation

▷ Could also ignore left subtree.

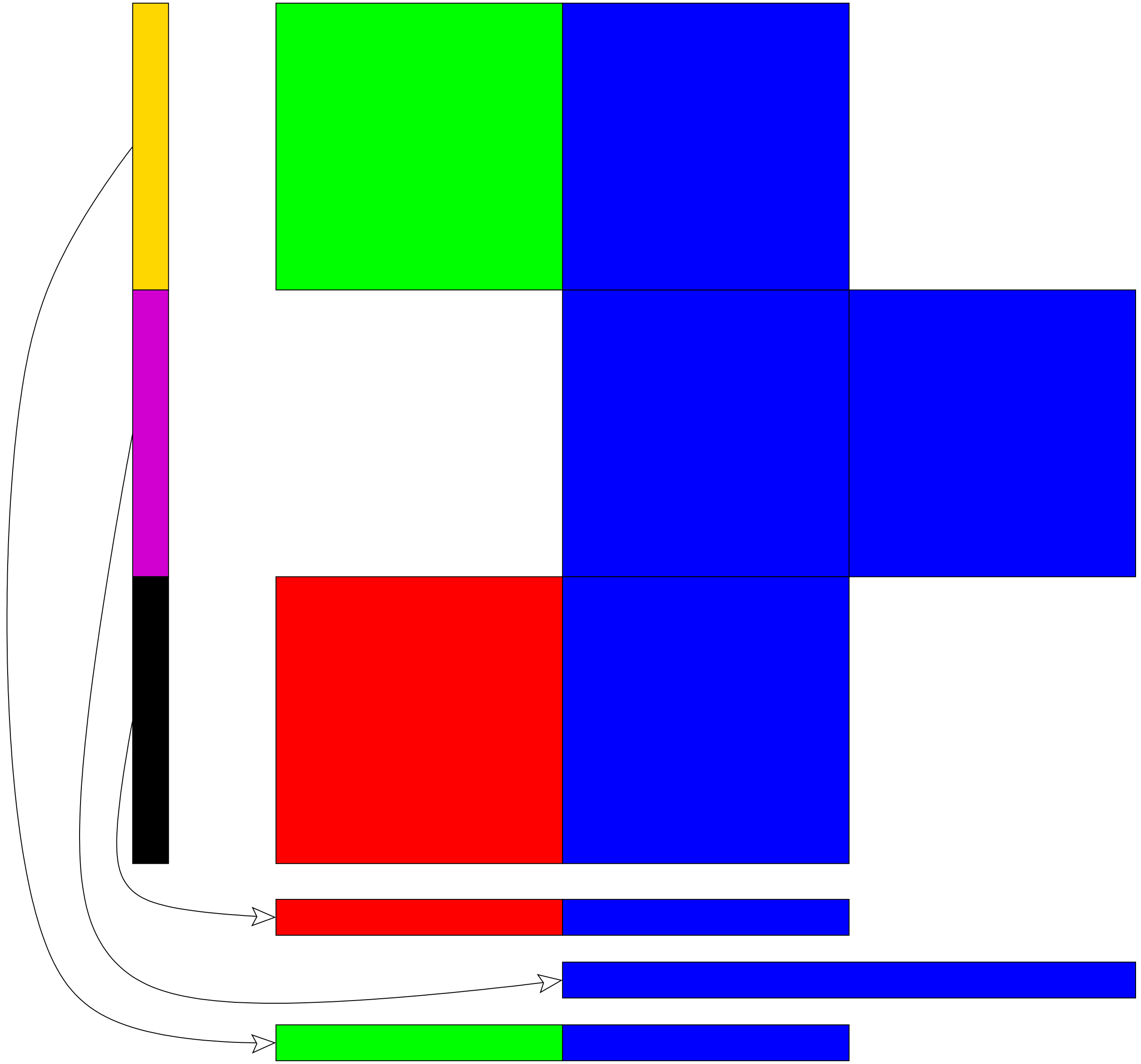


Compilation

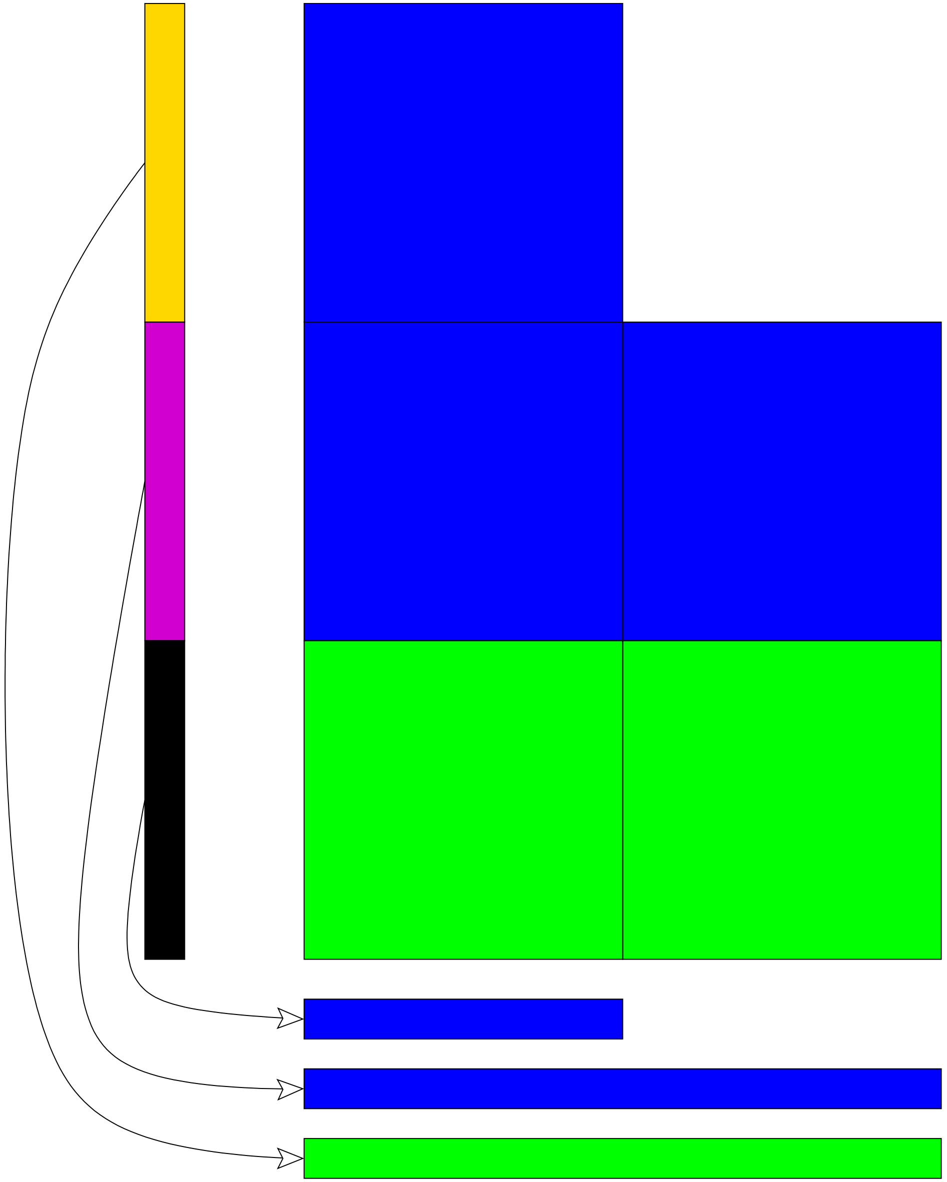
- Consider left subtree, but could ignore it.



Compilation



Compilation



Compilation of PM in CDuce

- Handle capture variables.
- All CDuce types (open/closed records, integer/characters intervals, . . .).
- Effectively encourages declarative style without degrading performances.

Thank you!