Counting in Trees for Free

Anca Muscholl

Joint work with:

Helmut Seidl, Thomas Schwentick, Peter Habermehl

TU Munich, Marburg, LIAFA & Univ. Paris 7

ICALP 2004

An e-donkey user:

```
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  <name> Charlie Brown </name>
  <email> cb@comics.org </email>
  <music> ... </music>
  <video> ... </video>
  <images> ... </images>
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 ...
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His downloaded music:

```
<jazz>
 <album> Always let me go </album>
 <artist> Keith Jarrett </artist>
 <year> 2002 </year>
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 <review> ... </review>
 <price> 24 </price>
</jazz>
<pop>
 <title> Just my imagination</title>
 <artist> The Cranberries </artist>
 <year> 2002 </year>
 <album> Stars </album>
 <price> 20 </price>
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</pop>
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```
<french>
 <title> Aux enfants de la chance </title>
 <artist> Serge Gainsbourg </artist>
 <album> Serge Gainsbourg, vol. 3 </album>
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 <review> ... </review>
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```
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 <review> ... </review>
 <review> ... </review>
 <review> ... </review>
</jazz>
```

... in essence:

→ hierarchically structured through tags;

```
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 <price> 22 </price>
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 <review> ... </review>
 <review> ... </review>
</jazz>
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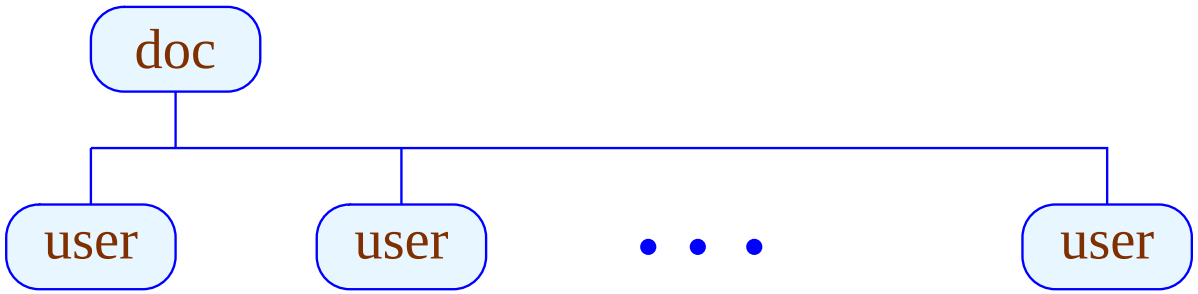
... in essence:

→ hierarchically structured through tags;

→ linear representation of (unranked) tree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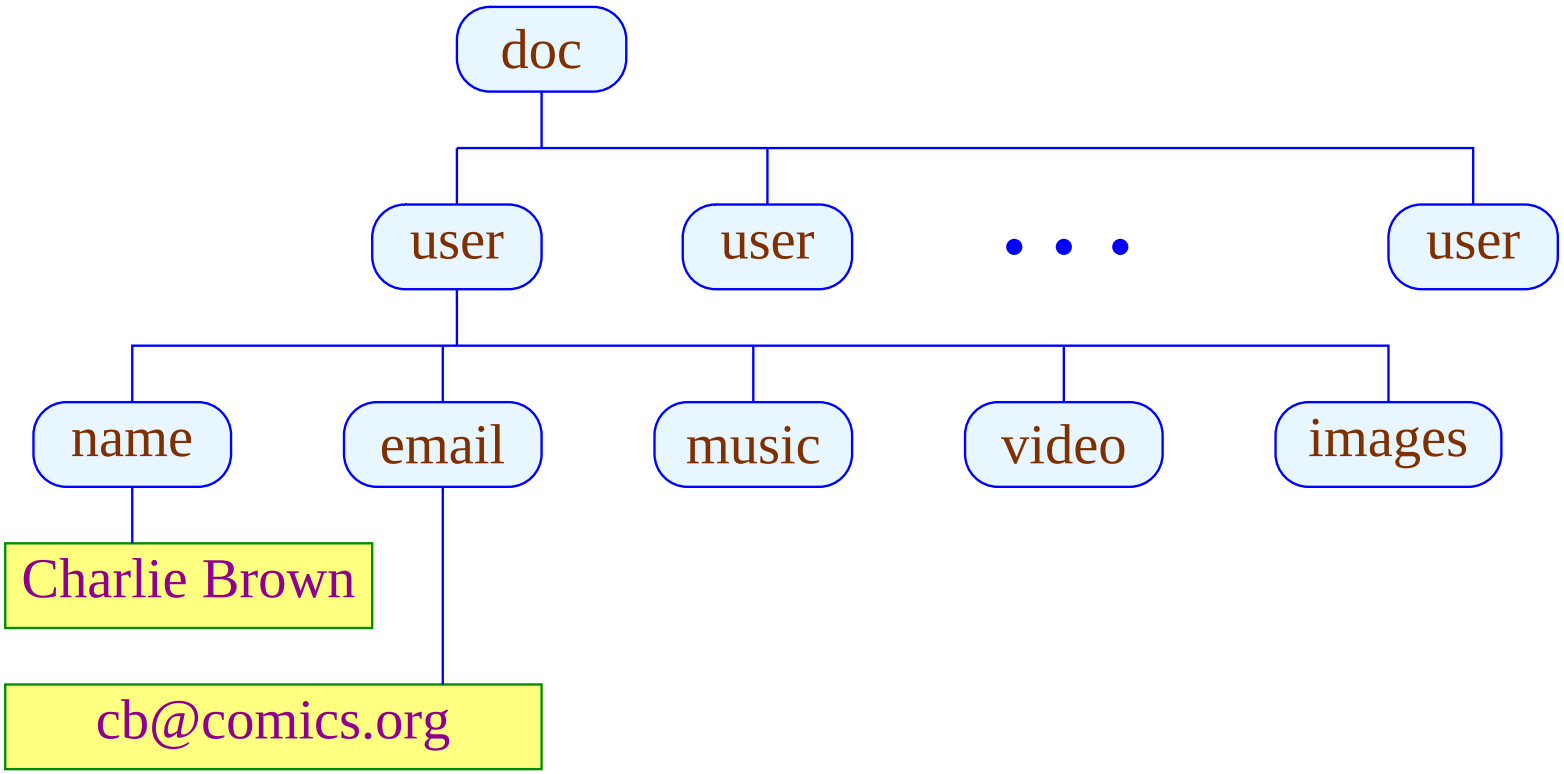
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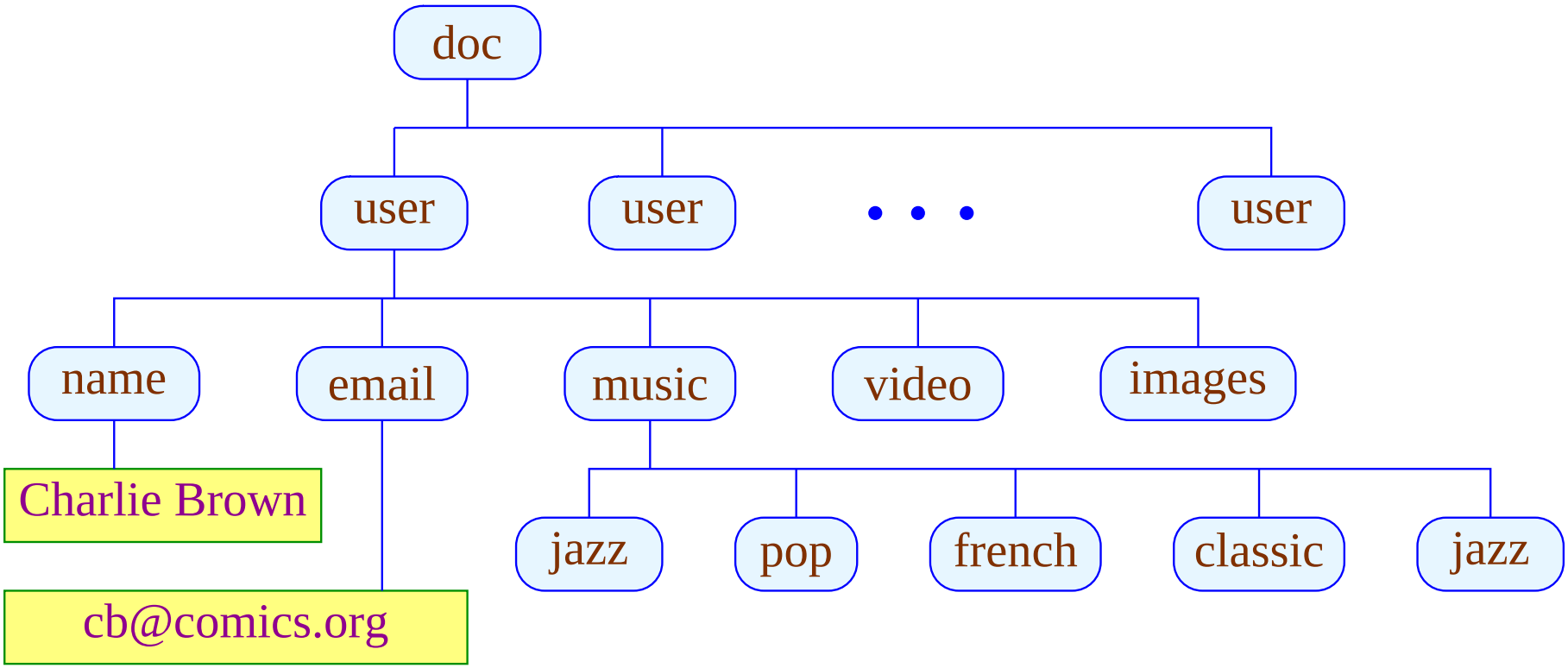
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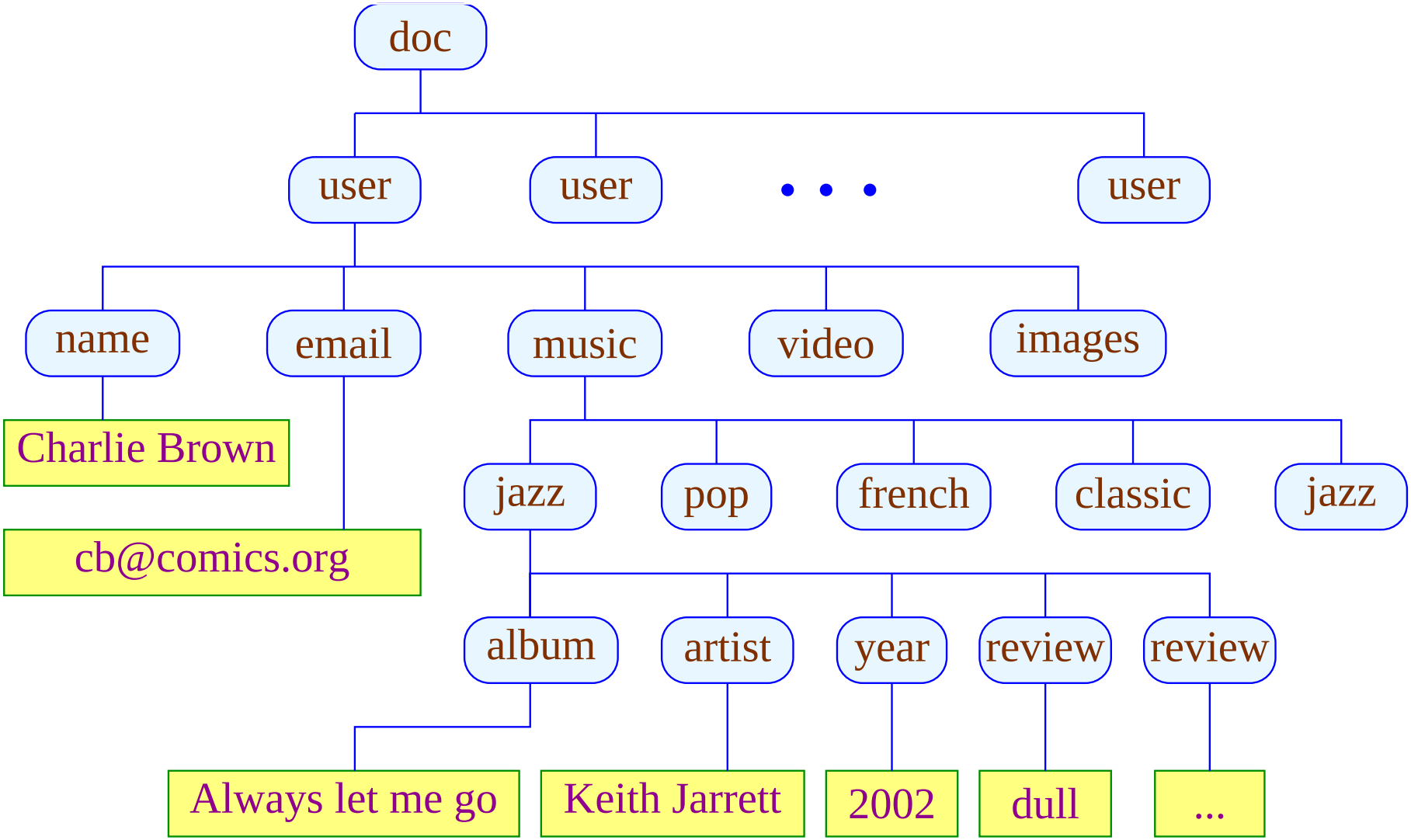
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Queries

Jazz with more than 2 reviews

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$$\text{jazz}\langle \#\text{review} > 2 \rangle$$

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Jazz with more than 2 reviews

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Users who like jazz more than pop

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Users who like jazz more than pop

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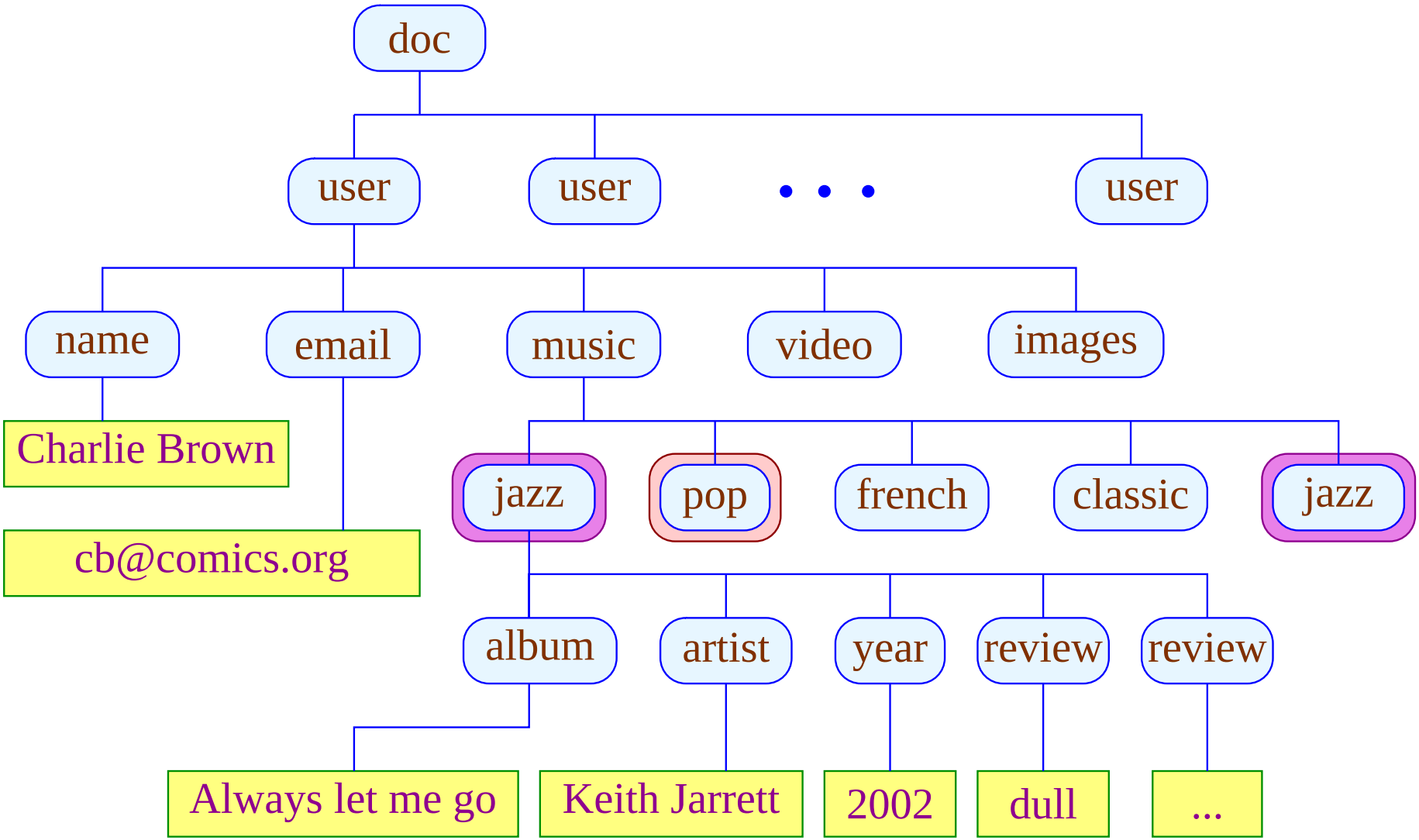
with

$$F = *\langle \#\text{jazz} > \#\text{pop} \rangle$$

(nodes with more jazz-children than pop-children)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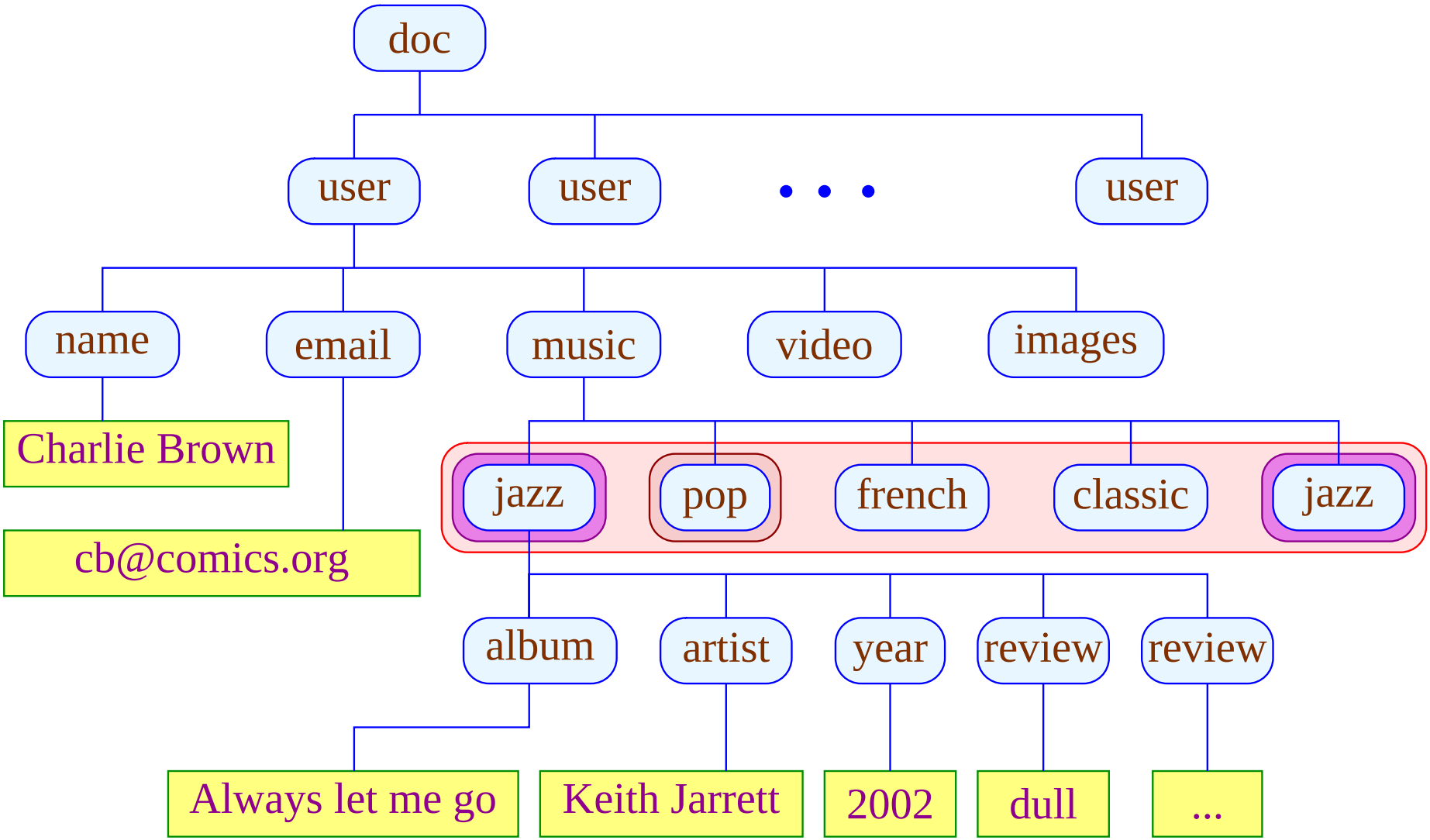
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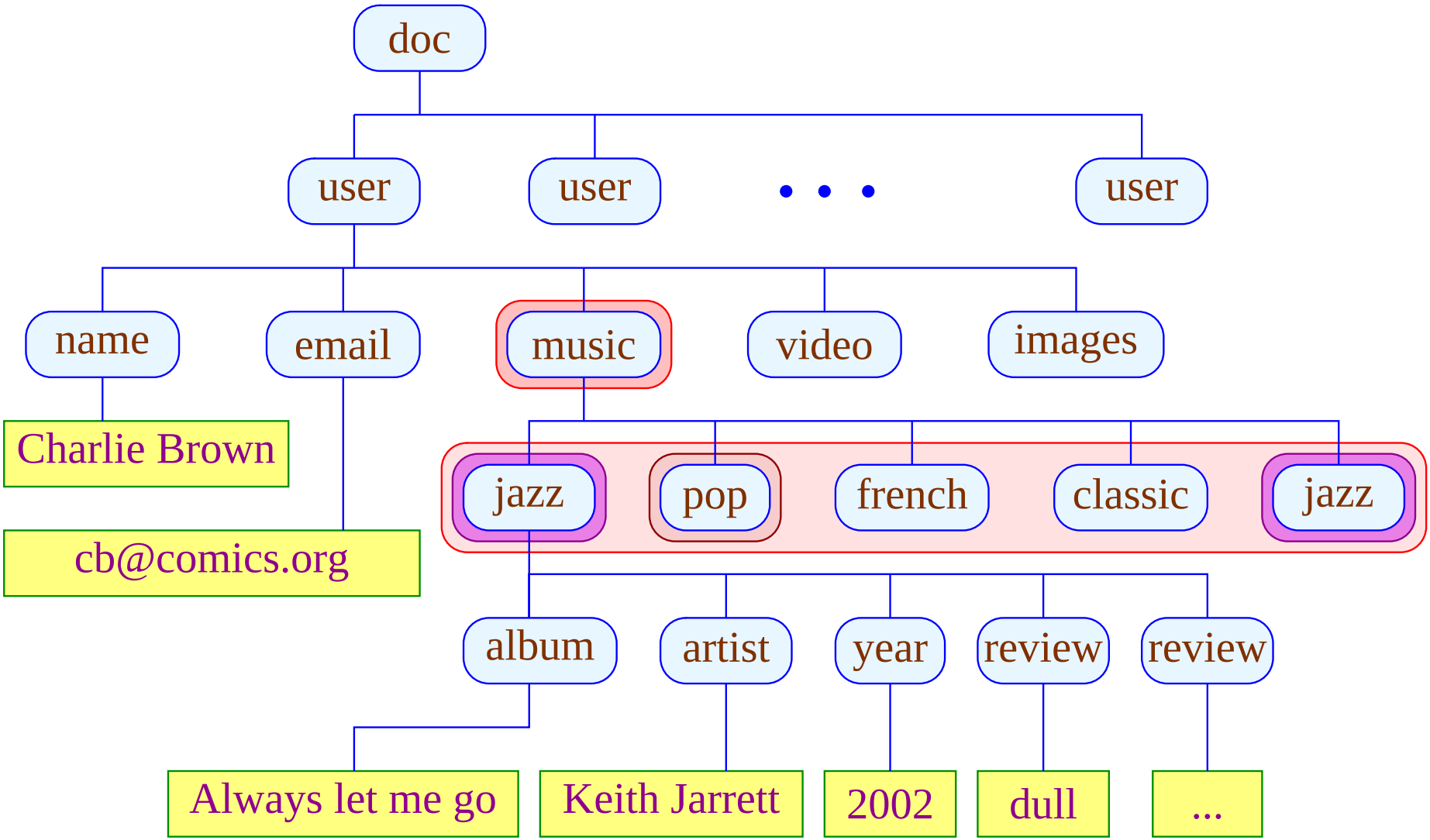
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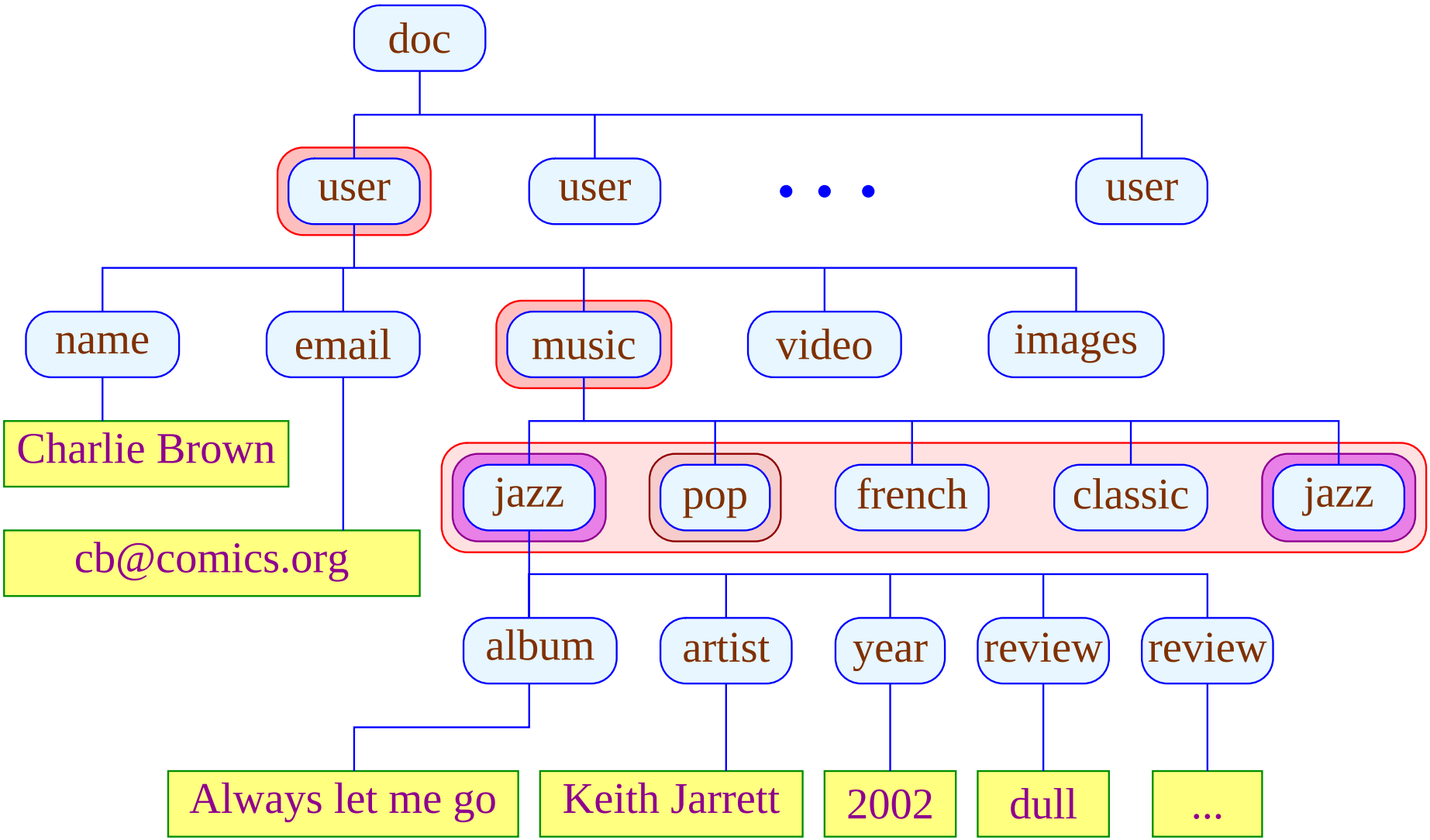
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Goal:

- → Extend query logics by numerical constraints on the children of nodes
- → Get implementation by extended tree automata

Fixpoint formulas:

$$\phi \quad ::= \quad \mathsf{tt} \mid \mathsf{ff} \mid x \mid \mu\, x.\, \phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid$$

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Precondition F: Boolean combination of regular expressions and Presburger constraints over formulas satisfied by children of node labeled a

Theorem

- $\mathcal{L} = \{t \mid t \models \phi\}$ for a Presburger fixpoint formula

  iff

  $\mathcal{L} = \mathcal{L}(A)$ for a deterministic Presburger automaton.
- Presburger fixpoint satisfiability is decidable in ExpTime.

Related Work:

Querying Ordered Documents:

XPath 2.0	W3C 2002
MSO	Schwentick et al. 2002

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Numerical Reasoning:

graded μ-calculus	Kupferman, Sattler, Vardi 2002
Sheaves Automata	Lugiez, Dal Zilio 2002
MSO + Presburger	Seidl, Schwentick, M. 2003

Presburger Arithmetic

$$\exists\, y.\ (5y \leq 3x \wedge 3x - 5y \leq 2)$$

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Formally,

$$\begin{aligned} \phi \quad ::= \quad & x_1 + x_2 = x_3 \mid x = n \\ & \mid \phi_1 \vee \phi_2 \mid \neg\phi \\ & \mid \exists\, x.\ \phi \end{aligned}$$

Satisfying Variable Assignments:

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$\rightarrow$ ... which can be decided for emptiness / membership

$\rightarrow$ ... by constructing a DFA A_ϕ

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(triple exponential, Klaedtke LICS 2004)

Presburger Tree Automata

Idea [S. et al. 2003 + Lugiez, Dal Zilio 2002]:

Allow as preconditions Boolean combinations of regular expressions and Presburger formulas.

Example: $q\ (q+p)^* \quad \wedge \quad \#p \geq 2(\#q) \quad \longrightarrow_a \quad s$

"Go to state s if node label is a, all children are in state p or q and there are at least twice as many in p, and the first child is in state q"

[Seidl et al. 2003]

For non-deterministic Presburger Tree Automata,

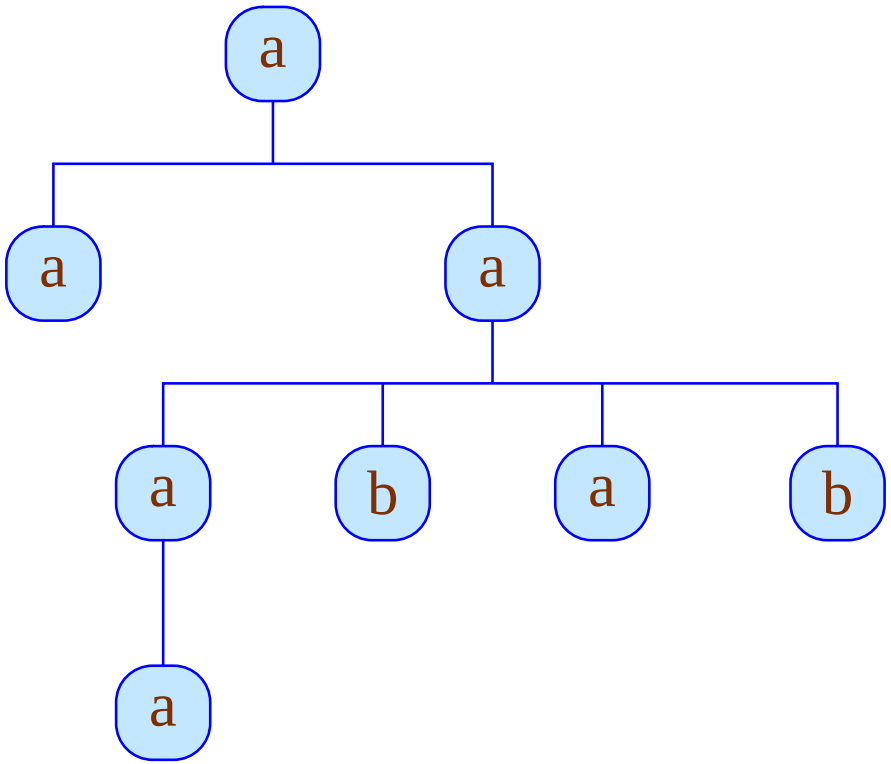
- Emptiness / membership are decidable;
- Universality is undecidable;
- Equivalence with Presburger EMSO (father/sibling + Presburger constraints on children)

Deterministic Presburger Tree Automata

Example:

Set of all trees over $\{a, b\}$ such that:

- Every internal node is labeled with a.
- The leftmost leaf is always labeled with a.
- The number of children with b-leaves is $\leq$ than the number of those with.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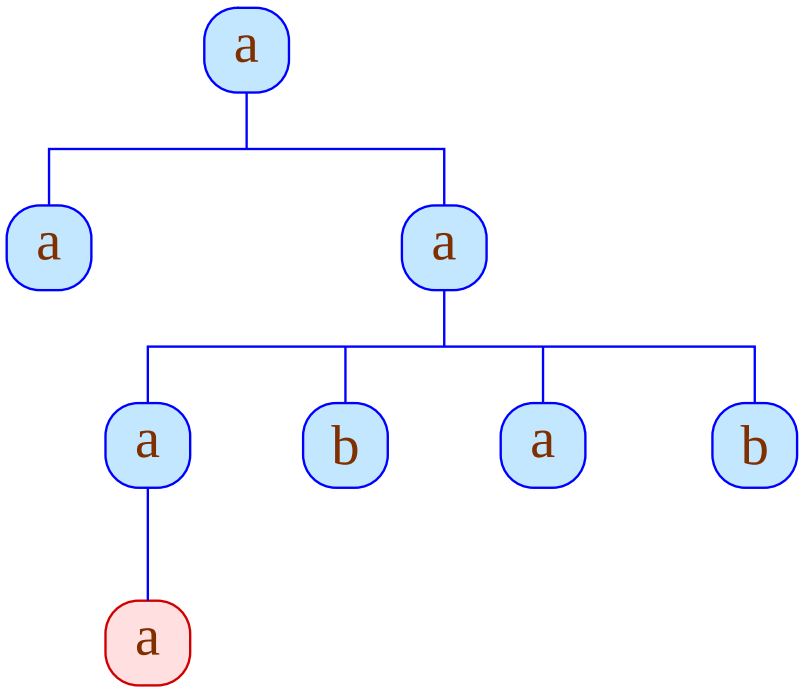
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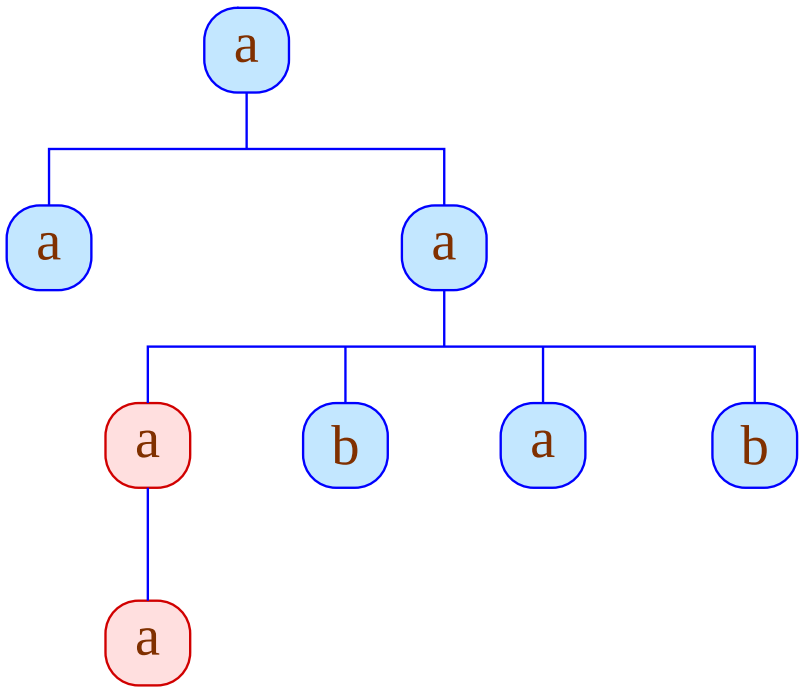
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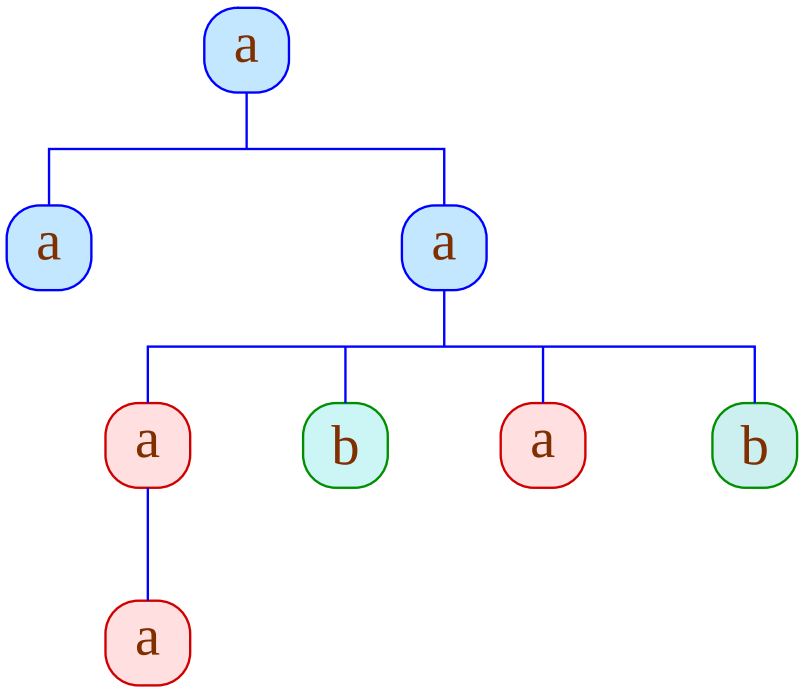
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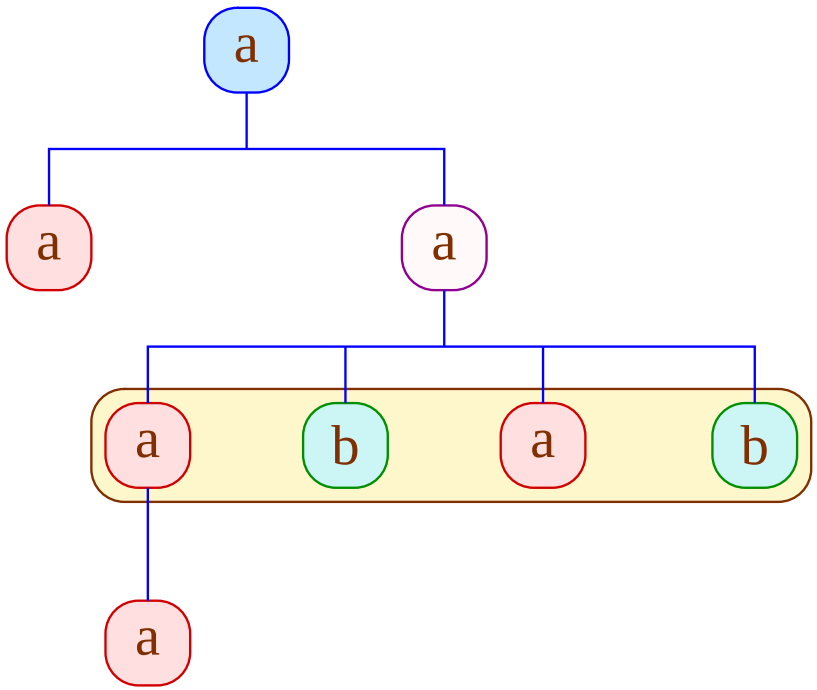
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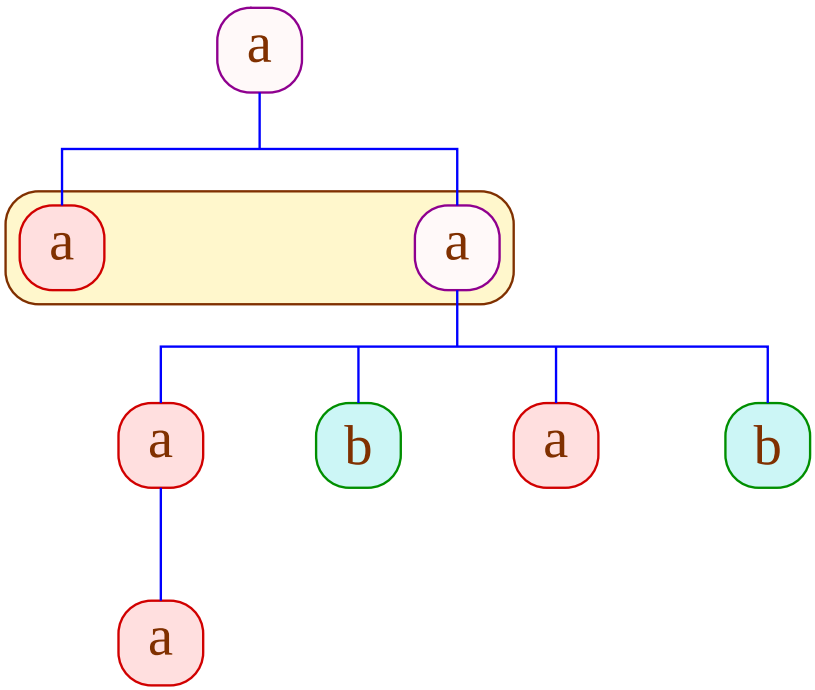
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Complexity...

Use quantifier-free Presburger formulas...

Theorem

Emptiness for PTA is PSPACE-complete.

Proof: Determine reachable states bottom-up.

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Transition precondition $\bigwedge_i R_i \wedge \bigwedge_j \overline{R'_j} \wedge \phi$: test emptiness

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- quantifier-free Presburger formulas... boolean expressions over equations with integer coefficients
- Parikh image of an NFA of size n over alphabet of size k is a semi-linear set over vectors from $\{1, \ldots, n\}^k$.

Complexity...

Parikh image of $\bigwedge_i R_i \wedge \bigwedge_j \overline{R'_j}$:

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[Papadimitriou'81] Size of minimal solution of diophantine system is only exponential in the number of equations

Complexity...

PTA Emptiness: Special case with better complexity

Presburger constraints $\bigvee_i (R_i \wedge \Phi_i)$, with Φ_i conjunction of equations

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Theorem

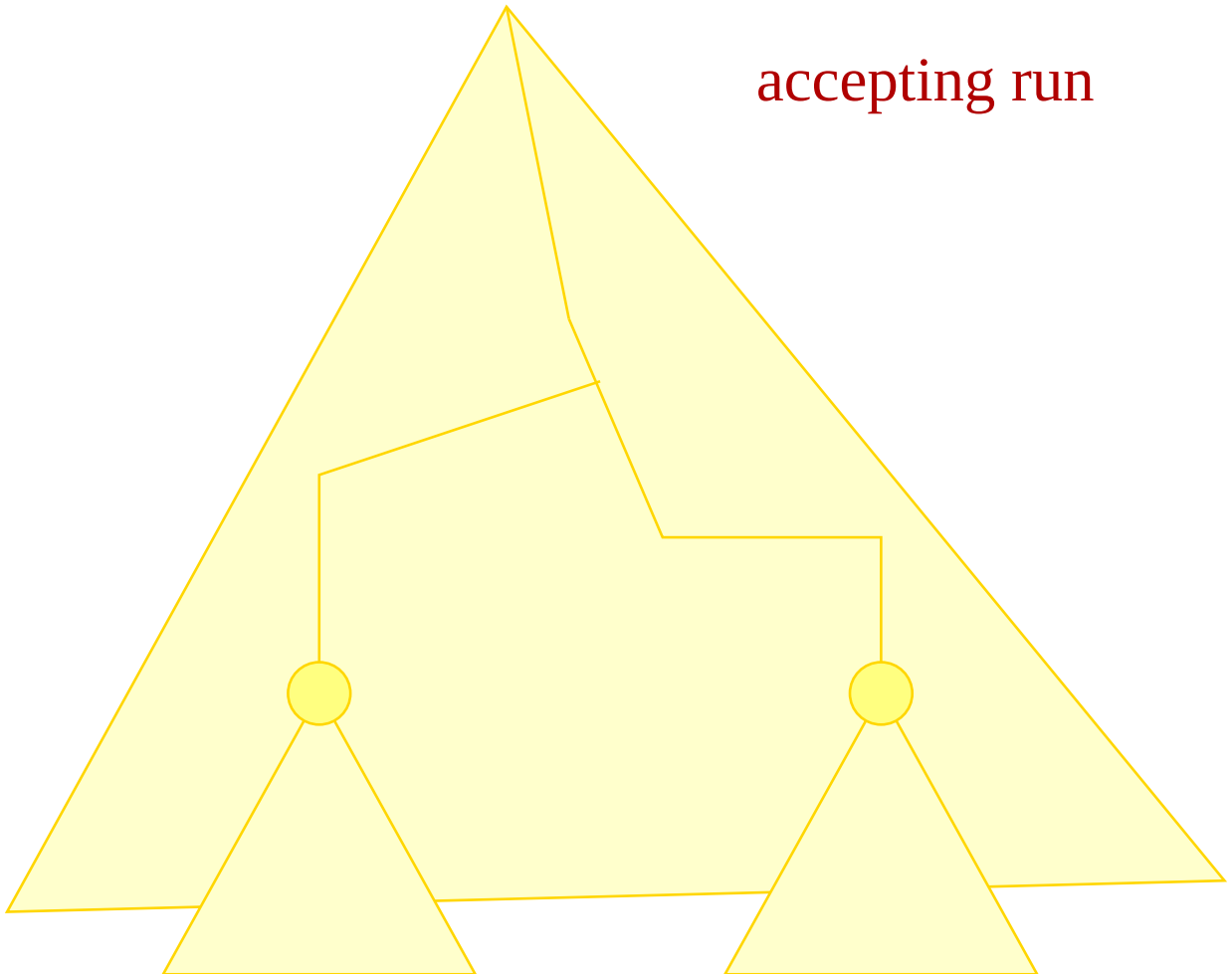
Membership $t \in \mathcal{L}(A)$ can be checked in time $O(|t||A|)$.

Presburger Fixpoint Query:

$\bullet \wedge \text{user} \wedge \mu x.\, (*\langle \_ x \_ \rangle \vee \text{jazz}\langle \#\text{review} > 2 \rangle)$

marker $\bullet$ for requested subdocument

Automata Query:



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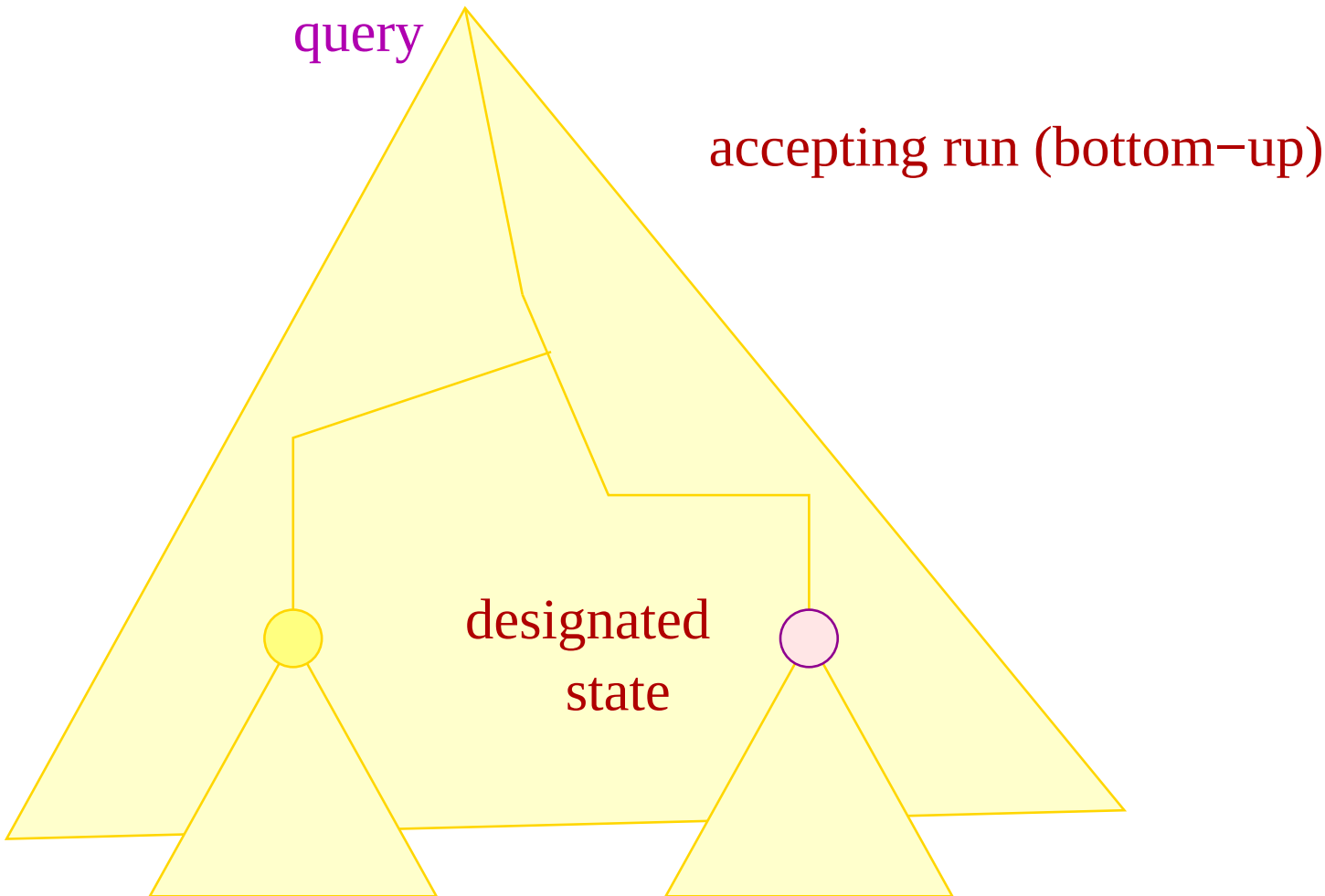

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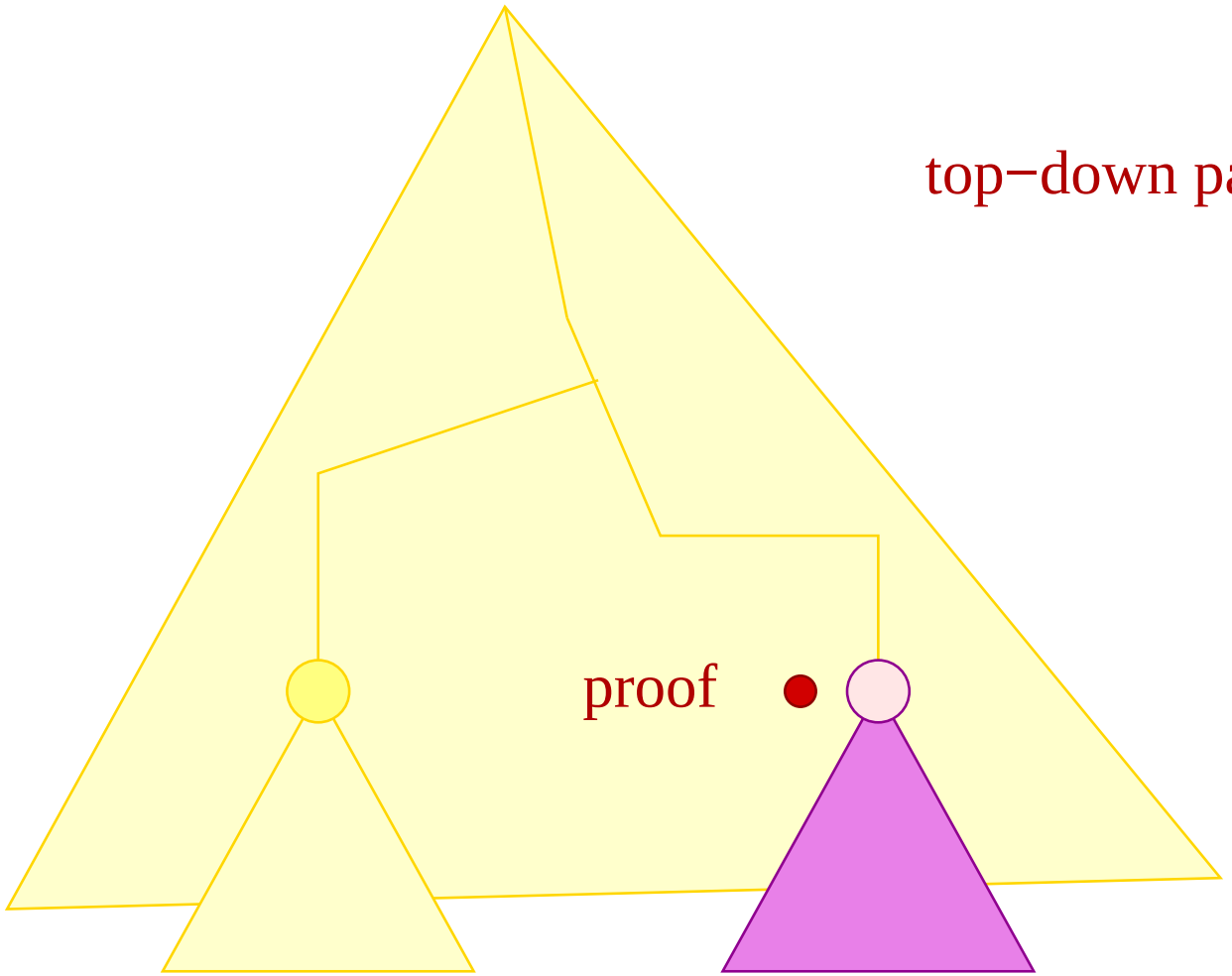

Theorem

A Presburger fixpoint query ϕ over a tree t can be answered in time $O(|t||\phi|^2)$.

Conclusion

→ Linear-time algorithm for computing the Parikh-image of an NFA.

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- → The technique can be generalized to reason about documents containing explicit natural numbers.

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→ The technique can be generalized to reason about documents containing explicit natural numbers.

→ Identify practical formalism for Presburger queries which supports:

- intuitive specifications;
- efficient compilation into automata.

Parikh images of regular sets

Theorem For an NFA A (or regular expression) an existential Presburger formula of linear size can be constructed for $\mathcal{L}(A)$.

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Proof.

Variables $x_{p,a,q}$ for each transition (p, a, q)

Formula describing connected flow = Parikh image of $w \in \mathcal{L}(A)$